

The Evolution of Diffuse Radio Sources in Galaxy Clusters

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ABSTRACT

We investigate the evolution and number distribution of radio halos in galaxy clusters. Without re-acceleration or regeneration, the relativistic electrons responsible for the diffuse radio emission will lose their energy via inverse-Compton and synchrotron losses in a rather short time, and radio halos will have lifetimes ~ 0.1 Gyr. Radio halos could last for \sim Gyr if a significant level of re-acceleration is involved. The lifetimes of radio halos would be comparable with the cosmological time if the radio-emitting electrons are mainly the secondary electrons generated by pion decay following proton-proton collisions between cosmic-ray protons and the thermal intra-cluster medium within the galaxy clusters. Adopting both observational and theoretical constraints for the formation of radio halos, we calculate the formation rates and the comoving number density of radio halos in the hierarchical clustering scheme. Comparing with observations, we find that the lifetimes of radio halos are \sim Gyr. Our results indicate that a significant level of re-acceleration is necessary for the observed radio halos and the secondary electrons may not be a dominant origin for radio halos.

Subject headings: cosmology: theory — galaxies: clusters: general — radio continuum: general

1. INTRODUCTION

Diffuse radio emission from galaxy clusters is a rare phenomenon. These radio sources, which usually possess large sizes and steep spectra, are called radio halos if they permeate the cluster centers and radio relics if they are located in cluster peripheral regions. Observations found that radio halos only exist in the clusters that show X-ray substructures (Feretti 2000). Since a galaxy cluster possessing X-ray substructures indicates that it is under ongoing merging, it is expected that the formation of radio halos is closely related to the merging process of galaxy clusters.

The diffuse radio emission of galaxy clusters is believed to be produced by the synchrotron radiation of relativistic electrons. Nonetheless, the sources of these relativistic electrons are still unclear. Cluster merging is a very violent event and releases a large amount of energy ($\sim 10^{64}$ ergs); this leads cluster mergers to be a very favorable mechanism for the production of the relativistic particles. However, relativistic electrons lose energy on the time scale of order $\sim 10^8$ years because of inverse Compton and synchrotron losses; this suggests that without re-acceleration radio halos in galaxy clusters might be transient features associated with a major merger and would have lifetimes ~ 0.1 Gyr (De Young 1992; Tribble 1993).

The numerical simulations of cluster mergers (e.g., Miniati et al. 2000; Ricker & Sarazin 2001) showed that the intra-cluster medium (ICM) is seriously disturbed by merging. Violent turbulence generated by mergers must play an important role in the re-acceleration of relativistic electrons (Sarazin 2001). Considering re-acceleration for the relativistic electrons, a two-phase model proposed by Brunetti et al. (2001) successfully reproduces the radial steepening of the spectral index, the radio spectrum steepening at high frequencies, and the HXR excess in the Coma cluster (see also Schlickeiser, Sievers, & Thiemann 1987; Liang, Dogial, & Birkinshaw 2002). On a similar study, Kuo, Hwang, & Ip (2003) showed that the “age” of Coma C might be ~ 1 Gyr; this indicates that the lifetimes of radio halos could be \sim Gyr if a significant level of re-acceleration is involved.

The secondary electron model first proposed by Dennison (1980) assumes that relativistic electrons are produced from the pion decay following collisions between the cosmic-ray protons and the thermal ions of the ICM. It has been recognized that the diffusion time of cosmic-ray protons is comparable with the cosmological time, so cosmic-ray protons are confined within galaxy clusters for the lifetimes of the clusters (Völk, Aharonian, & Breitschwerdt 1996; Berezhinsky, Blasi, & Ptuskin 1997; Schlickeiser, Sievers, & Thiemann 1987). If radio halos are formed from the secondary electrons (e.g., Blasi & Colafrancesco 1999; Miniati et al. 2001a), their lifetimes would be comparable with the cosmological time. The significantly different time scales of radio halos could have discernible effects on their number distribution and thus could discriminate on the origins of radio halos.

The formation rates of radio halos in galaxy clusters can be estimated from the hierarchical model. Press & Schechter (1974, hereafter PS) derived a mass function to evaluate the comoving number density of bound virialized objects, but this function does not specify the formation epochs of the objects. To solve this problem, Lacey & Cole (1993, hereafter LC) derived a distribution function of formation epochs by using the merger probabilities in the framework of PS formalism (see also Bond et al. 1991; Bower 1991). Based on the formalism of LC, Kitayama & Suto (1996a, hereafter KS) proposed another distribution function of

formation epochs in a similar but slightly different manner. We follow the formalism of KS but modify it to suit the situation for forming radio halos.

In this paper, we investigate the evolution and number distribution of radio halos with different lifetime scales. Three typical lifetimes that are representatives of different origins for radio halos are considered (i.e. 0.1 Gyr, 1 Gyr, and the cosmological time). We estimate the different number distributions and compare the results with observations to determine the valid models that are responsible for the origin of radio halos.

This paper is planned as follows. In § 2, we discuss the conditions for radio halos forming in galaxy clusters. In § 3, we describe the methods based on the formalism proposed by KS for calculating the formation rates and comoving number density of radio halos. In § 4, we compare the modelling results of radio halos with different lifetimes with observations, and in § 5 we give our discussion and conclusions.

2. FORMATION CRITERIA

Radio halos are not found in low X-ray luminosity clusters and only present in massive clusters with high X-ray luminosity and high temperature (Giovannini, Tordi, & Feretti 1999). This fact indicates that there might be a threshold mass for galaxy clusters to form radio halos. It has been recognized that radio halos are strongly correlated with cluster mergers (Feretti 2000). Nonetheless, many cluster mergers do not possess radio halos. Buote (2001) studied the dynamical states of clusters possessing radio halos and found that radio halos form preferentially in massive clusters experiencing violent mergers that have seriously disrupted the cluster core. Disrupting the cluster core in the merging process should be an important factor for forming radio halos. According to above discussion, we assume two criteria for a cluster merger to form a radio halo: (1) the cluster mass must be greater than or equal to a threshold mass, and (2) the merging process must be violent enough to disrupt the cluster core. Under these two conditions, mergers might generate sufficient primary electrons or secondary electrons from cosmic-ray protons to form radio halos.

We use observational data (Giovannini et al. 1999; Feretti 2000; Kempner & Sarazin 2001) to determine the threshold mass. We found that the A548b cluster has the lowest temperature ~ 2.4 keV (Giovannini et al. 1999), which is presumed to have the smallest mass from the well-known mass-temperature relation for galaxy clusters (e.g., Mulchaey 2000; Rosati, Borgani, & Norman 2002). Using the observed mass-temperature relationship

(Evrard, Metzler, & Navarro 1996; Horner, Mushotzky, & Scharf 1999)

$$M = 5 \times 10^{13} h^{-1} \left(\frac{T_X}{1 \text{keV}} \right)^{1.5} M_\odot, \quad (1)$$

we found that the A548b cluster has a mass of $\approx 2.7 \times 10^{14} M_\odot$ for $h = 0.7$ or $3.7 \times 10^{14} M_\odot$ for $h = 0.5$. Since very few clusters with lower temperatures have been found to possess radio halos, the threshold mass may be on the order of $\sim 10^{14} M_\odot$. We choose $10^{14} M_\odot$ to be the threshold mass M_{th} for cluster mergers to form radio halos. For comparison, we also consider different M_{th} ($5 \times 10^{13} M_\odot$ and $5 \times 10^{14} M_\odot$) in our calculation.

The condition for the disruption of cluster cores have been determined from dynamical simulation. Salvador-Solé, Solanes, & Manrique (1998) found that a cluster with mass M experiencing a merging process would disrupt its core structure when the relative mass increase $\Delta M/M$ exceeds a certain threshold $\Delta_m \equiv (\Delta M/M)_{\text{threshold}} = 0.6$. We use these two conditions, $M_{\text{th}} = 10^{14} M_\odot$ and $\Delta_m = 0.6$, as the criteria for cluster mergers to form radio halos.

3. FORMULATION

3.1. Formation Rates

The Press-Schechter mass function (PS) is used to model the cluster number density and its evolution. The comoving number density of clusters in the mass range $M \sim M + dM$ at time t is given by

$$n_{\text{ps}}(M, t) dM = \sqrt{\frac{2}{\pi}} \frac{\rho_0}{M} \frac{\delta_c(t)}{\sigma^2(M)} \left| \frac{d\sigma(M)}{dM} \right| \exp \left[-\frac{\delta_c^2(t)}{2\sigma^2(M)} \right] dM, \quad (2)$$

where ρ_0 is the present mean density of the universe, $\delta_c(t)$ is the critical density threshold for a spherical perturbation to collapse by the time t , and $\sigma(M)$ is the present rms density fluctuation smoothed over a region of mass M . We adopt the expressions of $\delta_c(t)$ summarized in Randall & Sarazin (2001) for different cosmological models. For $\sigma(M)$, we use an approximate formula proposed by Kitayama & Suto (1996b) for the cold dark model fluctuation spectrum and choose the value of σ_8 from the $\Omega_0 - \sigma_8$ constraint derived from the present cluster abundance: $\sigma_8 \Omega_0^{0.45} = 0.53$ (for $\Omega_\Lambda = 0$) and $\sigma_8 \Omega_0^{0.53} = 0.53$ (for $\Omega_0 + \Omega_\Lambda = 1$) (Pen 1998). The parameter $\Omega_0 \equiv \rho_0/\rho_c$ is the ratio of the present mean density to the critical density $\rho_c = 3H_0^2/(8\pi G)$ and $\Omega_\Lambda \equiv \Lambda/(3H_0^2)$, where Λ is the cosmological constant. The Hubble constant is defined as $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. Three different cold dark matter (CDM) models are considered in our analysis: a standard model (SCDM) ($\Omega_0 = 1, \Omega_\Lambda = 0, h = 0.5, \Gamma = 0.5$),

an open model (OCDM) ($\Omega_0 = 0.3, \Omega_\Lambda = 0, h = 0.7, \Gamma = 0.2$), and a low-density flat model (Λ CDM) ($\Omega_0 = 0.3, \Omega_\Lambda = 0.7, h = 0.7, \Gamma = 0.2$), where Γ is the shape parameter defined by Sugiyama (1995).

The formation rate for an object with mass M_f ($M_f > M_i$) formed from initial mass between M_i and $M_i + dM_i$ in unit time at t is given by (LC; KS)

$$r(M_i \rightarrow M_f; t) dM_i \equiv \frac{1}{\sqrt{2\pi}} \frac{1}{[\sigma^2(M_i) - \sigma^2(M_f)]^{3/2}} \left[-\frac{d\delta_c(t)}{dt} \right] \left| \frac{d\sigma^2(M_i)}{dM_i} \right| dM_i. \quad (3)$$

As mentioned in § 2, for a cluster merger with mass greater than the threshold mass M_{th} to form a radio halo, the cores of the progenitor sub-clusters must have been disrupted in the merging process. Assume a cluster of mass M ($M \geq M_{\text{th}}$) is formed from the merging process of two sub-clusters M_1 and M_2 , where $M_1 \geq M_2$. For M_1 to have significant core disruption during the merging process, M_2 have to be greater than some threshold mass, $M_2 \geq \Delta_m M_1$, according to Salvador-Solé et al. (1998). Since $M_1 \geq M_2$ and $M = M_1 + M_2 \geq (1 + \Delta_m) M_1$, we obtain the mass range of M_1 : $M/2 \leq M_1 \leq M/(1 + \Delta_m)$. The quantity M_2 can be the accumulated mass of the merged sub-clusters in multiple merging. The formation rate of radio halos in cluster mergers with mass $M \geq M_{\text{th}}$ at time t is given by

$$\begin{aligned} R_f(M, t) &= \int_{M_a}^{M_b} r(M' \rightarrow M; t) dM', \\ &= \sqrt{\frac{2}{\pi}} \left[-\frac{d\delta_c(t)}{dt} \right] \left[\frac{1}{[\sigma^2(M_b) - \sigma^2(M)]^{1/2}} - \frac{1}{[\sigma^2(M_a) - \sigma^2(M)]^{1/2}} \right], \end{aligned} \quad (4)$$

where $M_a = M/2$, and $M_b = M/(1 + \Delta_m)$.

3.2. Cumulative Comoving Number Density

The comoving number density of radio halos that form with cluster mass $M \sim M + dM$ at time $t_f \sim t_f + dt_f$ and survive until a latter time t is

$$\begin{aligned} n_{\text{rh}}(M, t_f, t) dM dt_f \\ = \begin{cases} n_{\text{ps}}(M, t_f) R_f(M, t_f) P_s(M, t_f, t) dM dt_f & \text{if } t_f \leq t \leq t_f + t_{\text{rh}}; \\ 0 & \text{if } t > t_f + t_{\text{rh}}, \end{cases} \end{aligned} \quad (5)$$

where P_s is the survival probability defined by KS,

$$P_s(M, t_1, t_2) = P(M' < (1 + \Delta_m)M, t_2 | M, t_1), \quad (6)$$

which is the probability that a cluster merger of mass M at t_1 will survive to have mass M' less than $(1 + \Delta_m)M$ at t_2 , and t_{rh} is the lifetime of radio halos. The lifetimes of radio halos may be slightly different from one another, but we ignore the deviation of the lifetime in calculation for simplicity. A survival merger increases its mass only by accretion without disrupting its core structure during the lifetime of its radio halo. We note that a radio halo might survive a new core-disruption merger even its host cluster are destroyed; the survival radio halo is treated as a new radio halo possessed by the new merger in our scheme.

The cumulative comoving number density of radio halos with cluster mass $\geq M(\geq M_{\text{th}})$ at time t can be evaluated as

$$n_{\text{rh}}(\geq M, t) = \int_M^\infty \int_{t-t_{\text{rh}}}^t n_{\text{rh}}(M', t_f, t) dM' dt_f \\ + \int_{M_L}^M \int_{t-t_{\text{rh}}}^t n_{\text{ps}}(M', t_f) R_f(M', t_f) P(M \leq M'' < (1 + \Delta_m)M', t | M', t_f) dM' dt_f, \quad (7)$$

where

$$P(M \leq M'' < (1 + \Delta_m)M', t | M', t_f) = P(M'' \geq M, t | M', t_f) - P(M'' \geq (1 + \Delta_m)M', t | M', t_f),$$

and

$$M_L = \begin{cases} M/(1 + \Delta_m) & \text{if } M > (1 + \Delta_m)M_{\text{th}}; \\ M_{\text{th}} & \text{if } M_{\text{th}} \leq M \leq (1 + \Delta_m)M_{\text{th}}. \end{cases}$$

The symbol M' is the cluster mass at t_f and M'' the cluster mass at t . If t_{rh} equals to the cosmological time, the term $t - t_{\text{rh}}$ in the time integral is replaced by 0. The second term in the right hand side of equation (7) represents the number density of radio halos that form with cluster mass less than M but increase their mass to greater than M by accretion at the time t .

4. RESULTS AND COMPARISONS WITH OBSERVATIONS

The evolution of the total comoving number density of radio halos is shown in Figure 1. We note that there is a maximum at $z \sim 0.4 - 0.5$ in the evolution of the total number density of radio halos for clusters with mass $M \geq 10^{14} M_\odot$ with $t_{\text{rh}} = 0.1$ Gyr and 1 Gyr in the Λ CDM and OCDM models; there is no such distribution maximum if we consider only the clusters with mass $M \geq 10^{15} M_\odot$. This may represent that before the period, the formation of many clusters with mass $\geq 10^{14} M_\odot$ are significantly due to violent merger of subclusters with mass $< 10^{14} M_\odot$; after the period, the violent-merger rates are very small and the formation of the clusters with mass $\geq 10^{14} M_\odot$ are mainly due to accretion of subclusters with mass

$< 10^{14} M_{\odot}$. But many massive clusters ($M \geq 10^{15} M_{\odot}$) are still mainly due to violent merger of subclusters.

Giovannini et al. (1999) searched radio halo candidates in the NRAO VLA Sky Survey from a sample of X-ray bright clusters presented by Ebeling et al. (1996). They found that the percentage of galaxy clusters possessing diffuse radio sources is $6\% \sim 9\%$ for $L_X \leq 10^{45}$ erg s $^{-1}$ and $27\% \sim 44\%$ for $L_X > 10^{45}$ erg s $^{-1}$. Here L_X is the luminosity in the 0.1–2.4 keV energy band. The redshift distributions of radio halos and the parent X-ray cluster population inspected by Giovannini et al. (1999) are shown in Figure 2. In this figure, We have corrected two uncertain halos (A754 and A2219) as confirmed ones and eliminated the uncertain halo in A2390 which is found to be a mini-halo (Bacchi et al. 2003). For $L_X > 10^{45}$ erg s $^{-1}$, the corrected percentage of galaxy clusters possessing diffuse radio sources is $28\% \sim 41\%$.

In Table 1, we show the number density ratios of radio halos to clusters in different cosmological models with different lifetimes of radio halos. To compare with the observations, we divide the mass of clusters possessing radio halos into two ranges, $M_{\text{th}} \leq M < 10^{15} M_{\odot}$ and $M \geq 10^{15} M_{\odot}$. The mass $M \sim 10^{15} M_{\odot}$ roughly corresponds to the luminosity $L_X \sim 10^{45}$ erg s $^{-1}$ (e.g., Rosati et al. 2002). We note that the ratios are a function of redshift and the percentages in Table 1 are estimated at low $z \leq 0.4$ so that they can be compared with the observations. Obviously, the theoretical percentages of radio halos with $t_{\text{rh}} = 0.1$ Gyr are much lower than the observational results for $L_X > 10^{45}$ erg s $^{-1}$. In other words, we would expect to observe much less radio halos in high X-ray luminosity clusters if radio halos were transient phenomena with lifetimes ~ 0.1 Gyr. On the other hand, the expected percentages are much higher than observational results for those radio halos with $t_{\text{rh}} = \text{cosmological time}$; we would have observed much more radio halos if radio halos had the cosmological lifetime. We find that only radio halos with $t_{\text{rh}} = 1$ Gyr can produce results roughly matching the observations in two mass ranges. Because of the limits of present instruments, the detection of radio halos in high-luminosity clusters is easier than that in low-luminosity ones. Thus the percentage of radio halos for $L_X > 10^{45}$ erg s $^{-1}$, $28\% \sim 38\%$, may be more robust than that for $L_X \leq 10^{45}$ erg s $^{-1}$. We note that different cosmological models have some effects on the calculated ratios but the effects are too small to cause confusion.

In Figures 3–5, we show the redshift distributions of the ratios of radio halos to the galaxy clusters. It is obvious that in all three different cosmological models only the results with the lifetime $t_{\text{rh}} = 1$ Gyr roughly fit the observations at $z \leq 0.18$. We note that the number of the cluster sample for $z > 0.18$ are too small and this may result in a large deviation. In particular, people tend to observe only high luminous clusters at high redshifts and the ratio of radio halos is higher in high luminous clusters; the high ratios observed at

$z > 0.18$ thus do not represent the real ratio of radio halos to the total clusters.

In Figure 6, we show the evolution of the ratios up to $z = 2$. For simplicity, we only show the results of the Λ CDM models; other cosmological models show similar results. We find that the ratios of the model with cosmological lifetime always increase, the model of Gyr lifetime show a distribution peak at $z = 1.6$, and the ratios of the 0.1 Gyr models decrease as the universe evolves. These results can also be used as an indicator of the origin of the radio sources if we have precise measurements and statistics for clusters at high redshifts.

To investigate the effects of the threshold mass M_{th} , we have adopted different M_{th} on our models. In Table 2, we show the ratios of radio halos to the galaxy clusters in the mass range $M_{\text{th}} \leq M < 10^{15} M_{\odot}$ for $M_{\text{th}} = 5 \times 10^{13} M_{\odot}$ and $5 \times 10^{14} M_{\odot}$ at $z \leq 0.4$. The ratios for clusters with mass $\geq 10^{15} M_{\odot}$ are not affected by the low mass threshold. We find that the ratio distributions of $M_{\text{th}} = 5 \times 10^{13} M_{\odot}$ and $5 \times 10^{14} M_{\odot}$ are not very different from that of $M_{\text{th}} = 10^{14} M_{\odot}$.

5. DISCUSSION AND CONCLUSIONS

The distribution of galaxy clusters possessing radio halos provides a strong constraint on the origins of radio halos. About 28% \sim 38% of clusters with $L_X > 10^{45} \text{ erg s}^{-1}$ possess radio halos. Since these X-ray luminous clusters are generally very massive and have undergone through multiple mergers during their formation, it is expected that these massive clusters would have accumulated a large amount of cosmic-ray protons from their formation history (e.g., Miniati et al. 2001b). The cosmic-ray protons would be confined in the clusters longer than the cosmological time, and the lifetimes of radio halos would be comparable with the cosmological time in the secondary electron model. If the secondary electron model was applicable, most of these massive clusters should possess radio halos; our results show that the percentage of clusters possessing radio halos should be greater than 70%. This is inconsistent with the observations in which only \sim 35% of these massive clusters possess radio halos. According to these results, the secondary electrons do not seem to be the dominant origin of the radio halos.

On the other hand, if radio halos were transient phenomena associated with a single acceleration event, such as a major merger shock, they would have lifetimes \sim 0.1 Gyr. Because of the short lifetimes of the sources, radio halos would be hardly observable even in the massive clusters. The observed percentage \sim 35% is thus too high to explain in the hierarchical clustering formation model.

According to the results presented in § 4, the lifetimes of radio halos may be \sim Gyr.

As mentioned in § 1, relativistic electrons in ICM lose energy on the time scale of order $\sim 10^8$ years because of the inverse Compton and synchrotron losses. This indicates that a significant level of re-acceleration is necessary to support the relativistic electrons against radiative losses and to maintain radio halos to last for \sim Gyr (Brunetti et al. 2001; Kuo et al. 2003).

As discussed in § 4, the percentage of radio halos for $L_X > 10^{45}$ erg s $^{-1}$, $\sim 28\%$ – 38% , is more robust. We here investigate the effects of the lifetime of the radio halos, the dividing mass, and the threshold mass ratio Δ_m on our results. Only the Λ CDM model is considered. First, the lifetimes of radio halos can affect the ratios of the radio halos. The percentage of radio halos with $t_{\text{rh}} = 1$ Gyr is $\sim 21\%$ and seems to be lower than the observational results, $\sim 28\%$ – 38% , as shown in Table 1. However, we note that the lifetime of the radio halo is of \sim Gyr and could be slightly longer or shorter than 1 Gyr and the difference of the assumed lifetime could affect the predicted percentage of the radio halo. For example, if a longer lifetime for the radio halo, $t_{\text{rh}} = 1.5$ Gyr, is assumed, then the percentage of the radio halo would become $\sim 32\%$ and would be in agreement with the observational results. Second, the dividing mass at $M = 10^{15} M_\odot$ may be lower than the realistic mass corresponding to $L_X = 10^{45}$ erg s $^{-1}$. For example, the A773 cluster with $L_X \sim 1.01 \times 10^{45}$ erg s $^{-1}$ has mass in the range 1.25 – $2.08 \times 10^{15} M_\odot$ (Govoni et al. 2001). In Table 3, we show the percentages corresponding to different dividing masses. Obviously, the results with $t_{\text{rh}} = 1$ Gyr are improved but those with $t_{\text{rh}} =$ cosmological time are worse if a higher and more realistic dividing mass is taken. Third, different threshold Δ_m also affect our results. The results with different Δ_m are shown in Table 4. The values of Δ_m strongly affect the results for $t_{\text{rh}} = 1$ Gyr and $t_{\text{rh}} =$ cosmological time. For $\Delta_m = 0.7$, the results with $t_{\text{rh}} =$ cosmological time seems to be close to the observational results; this implies that the radio halos would only be generated in the mergers with two nearly equal-mass progenitors if the secondary electrons were the dominant origin for forming radio halos. For $\Delta_m = 0.5$, the ratio of the radio halos with $t_{\text{rh}} = 1$ Gyr is consistent with the observational results. Obviously, the Δ_m is a dominant factor in the determining the ratio of the radio halos. Thus it is important to investigate in detail the exact value of Δ_m to clarify the question.

Note that we have assumed that a merger with mass greater than a threshold mass will form a radio halo if its relative mass increase exceeds a threshold, Δ_m . It might be possible that some mergers that satisfy our criteria did not form radio halos. However, according to the study of Buote (2001) for ~ 30 bright X-ray clusters, the number of the unaccounted mergers should be very small and thus has little effect on our results.

Magnetic fields could be also an important parameter in determining the radio powers of the cluster radio halos. It is very difficult and complicated to determine this parameter.

Magnetic fields $\leq 0.4 \mu\text{G}$ were found for several clusters using the inverse Compton models for the hard X-ray excess (Fusco-Femiano et al. 2003). Clarke et al. (2001) found that the cluster fields are typically around $5\text{--}10 \mu\text{G}$ using rotation measurements. Furthermore, during a cluster merging process, the magnetic fields might be amplified by a factor of ~ 20 on small scales (Roettiger, Stone, & Burns 1999) and might have a non-negligible effect on the radio emission. However, we note that both the magnetic field energy and the relativistic particle energy are provided by the merging energy, which is a function of the merger mass. In other words, the cluster mass should be a more fundamental parameter in determining the radio powers of the radio halos (Buote 2001; Govoni et al. 2001). Since we have considered the merger mass as a main parameter in our calculation, we thus do not treat the magnetic field as an independent parameter.

We could try to match the results obtained from the secondary electron model ($\sim 70\%$) with the observations ($\sim 35\%$) by assuming that about half of the radio halos produced by the secondary electrons are unobservable. However, Miniati et al. (2001a) showed that for a cluster of given mass the radio power from the secondary electrons could vary by almost an order of magnitude in their simulation. We note that such level of variation, as also detected in observation (Bacchi et al. 2003), is not large enough to make a detectable radio halo source to become non-detectable, especially for high X-ray luminosity clusters at low redshifts. It is thus unlikely to apply some observational effects to lower the radio halo fraction of the secondary electron model to match the observational results.

The radio halo fraction of galaxy clusters have been estimated in some recent studies. Fujita & Sarazin (2001) estimated the fraction of cluster radio halos based on the radiative energy loss timescale of the relativistic electrons. This timescale is similar to our case for primary electrons without re-acceleration and can account for only $\sim 10\%$ of observations. To match the observations, they assumed that even a rather weak merger should also trigger a radio halo. This assumption seems to be in contradiction with observations, which showed that only massive clusters experiencing violent mergers can have halos (Buote 2001) as we have discussed in § 2. Enßlin & Röttgering (2002) estimated the cluster radio halo luminosity function by assuming a radio halo fraction $f_{\text{rh}} = 1/3$ for all clusters. However, observations showed that the halo fraction is very different for low and high luminosity X-ray clusters and it is inconsistent with current observations to assume a constant radio halo fraction for all clusters. Giovannini et al. (1999) have noted that the lack of diffuse radio sources in low X-ray luminosity clusters is real because of their low redshifts; Bacchi et al. (2003) have also stressed that the correlation between the halo radio power and the cluster X-ray luminosity is only applied to clusters showing major mergers and therefore cannot be generalized to all clusters. The other more realistic fraction adopted by Enßlin & Röttgering (2002) is estimated by assuming that a cluster can possess a radio halo if its mass increases by more

than 40% of its present mass within half a dynamical timescale. In their work, the fraction for clusters with mass of $10^{15}M_{\odot}$ is 0.32. This seems to agree with the observational results for $L_X > 10^{45} \text{ erg s}^{-1}$. However, the fraction for clusters with mass of $10^{14}M_{\odot}$ and that for clusters with mass of $10^{13}M_{\odot}$ are 0.26 and 0.22, respectively. These results indicate that the radio halo ratio for $L_X \leq 10^{45} \text{ erg s}^{-1}$ should be greater than 20% and therefore are in conflict with the observations. We also note that the effects of the secondary electron model on the radio halo fraction were not considered in previous studies.

Our results are based on the assumption that the diffuse radio emission is associated with cluster merger events. We note that radio galaxies, starbursts, and AGNs might also contribute significant relativistic electrons to the ICM of the clusters. However, the high-energy ($\geq 1 \text{ GeV}$) relativistic electrons injected by these sources would become invisible in a very short time and can not directly account for the extended radio emission. On the other hand, the low-energy ($\leq 1 \text{ GeV}$) relativistic electrons can survive a longer time and become more extended via diffusion. A cluster merger event, as suggested in this paper for the diffuse radio emission phenomena, can produce merger shocks and MHD turbulence that can re-accelerate the survived low-energy relativistic electrons and reignite the radio emission in a more extended region. In this respect, our results are also applicable even the relativistic electrons are originally injected from radio galaxies, starburst galaxies, or AGNs.

We note that the effects of the inverse Compton loss rising with increasing redshift may reduce the ratio of radio halos at high redshifts, but the results at low redshifts are only slightly affected and the conclusions are still tenable.

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Table 1. The Ratios of Galaxy Clusters Containing Diffuse Radio Sources

Observations	$L_X \leq 10^{45} \text{ erg s}^{-1}$			$L_X > 10^{45} \text{ erg s}^{-1}$		
Halos	$\sim 4\%$			$\sim 28\text{--}38\%$		
Halos+Relics	$\sim 6\text{--}9\%$			$\sim 28\text{--}41\%$		
Models	$M_{\text{th}} \leq M < 10^{15} M_{\odot}$			$M \geq 10^{15} M_{\odot}$		
	SCDM	OCDM	Λ CDM	SCDM	OCDM	Λ CDM
0.1 Gyr	$\sim 2\%$	$\sim 1\%$	$\sim 1\%$	$\sim 4\%$	$\sim 2\%$	$\sim 2\%$
1 Gyr	$\sim 14\%$	$\sim 10\%$	$\sim 9\%$	$\sim 38\%$	$\sim 24\%$	$\sim 21\%$
cosmological time	$\sim 38\%$	$\sim 40\%$	$\sim 37\%$	$\sim 80\%$	$\sim 77\%$	$\sim 70\%$

Table 2. The Ratios of Galaxy Clusters Containing Diffuse Radio Sources in the Mass Range $M_{\text{th}} \leq M < 10^{15} M_{\odot}$ Under Different M_{th}

M_{th}	$5 \times 10^{14} M_{\odot}$			$5 \times 10^{13} M_{\odot}$		
	SCDM	OCDM	Λ CDM	SCDM	OCDM	Λ CDM
0.1 Gyr	$\sim 3\%$	$\sim 2\%$	$\sim 2\%$	$\sim 1\%$	$\sim 1\%$	$\sim 1\%$
1 Gyr	$\sim 22\%$	$\sim 16\%$	$\sim 14\%$	$\sim 12\%$	$\sim 8\%$	$\sim 8\%$
cosmological time	$\sim 39\%$	$\sim 41\%$	$\sim 38\%$	$\sim 35\%$	$\sim 37\%$	$\sim 34\%$

Table 3. The Ratios of Galaxy Clusters Containing Diffuse Radio Sources in the Mass Range $M \geq M'$ in Λ CDM

M'	$1 \times 10^{15} M_{\odot}$	$1.2 \times 10^{15} M_{\odot}$	$1.5 \times 10^{15} M_{\odot}$	$2 \times 10^{15} M_{\odot}$
0.1 Gyr	$\sim 2\%$	$\sim 3\%$	$\sim 3\%$	$\sim 3\%$
1 Gyr	$\sim 21\%$	$\sim 23\%$	$\sim 25\%$	$\sim 29\%$
cosmological time	$\sim 70\%$	$\sim 73\%$	$\sim 77\%$	$\sim 82\%$

Table 4. The Ratios of Galaxy Clusters Containing Diffuse Radio Sources for Different Δ_m in the Mass Range $M \geq 10^{15} M_{\odot}$ in Λ CDM

Δ_m	0.5	0.6	0.7
0.1 Gyr	$\sim 3\%$	$\sim 2\%$	$\sim 2\%$
1 Gyr	$\sim 32\%$	$\sim 21\%$	$\sim 14\%$
cosmological time	$\sim 95\%$	$\sim 70\%$	$\sim 49\%$

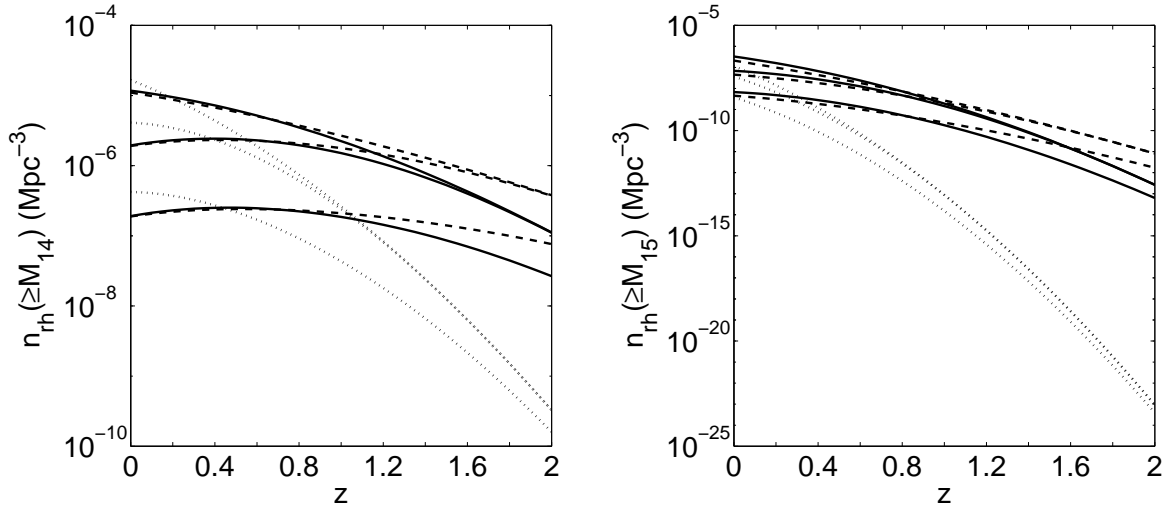


Fig. 1.— (a) The evolution of the total number density of radio halos with cluster mass $M \geq M_{14}$, where $M_{14} = 10^{14} M_{\odot}$. Three different cosmological models, SCDM (*dotted curves*), OCDM (*dashed curves*), and Λ CDM (*solid curves*) with three representative lifetimes of radio halos: 0.1 Gyr, 1 Gyr, and the cosmological time (*bottom to top*) are shown. (b) Same as (a), but for $M \geq 10^{15} M_{\odot}$.

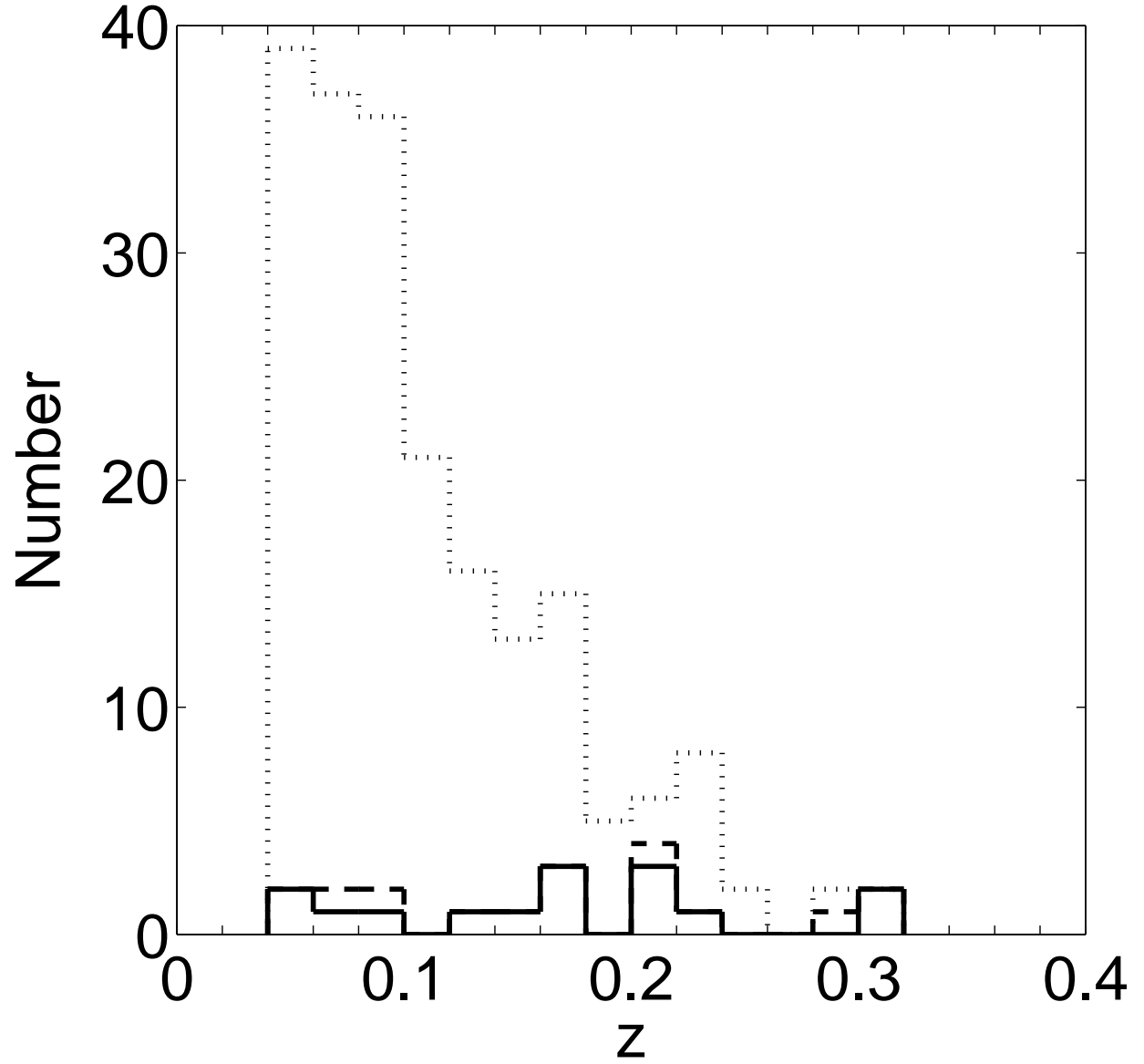


Fig. 2.— The distributions of the X-ray clusters inspected (*dotted line*) and radio halos detected (*solid line* and *dashed line*) by Giovannini et al. (1999). The *dashed line* includes halo sources that have uncertainty in their detection.

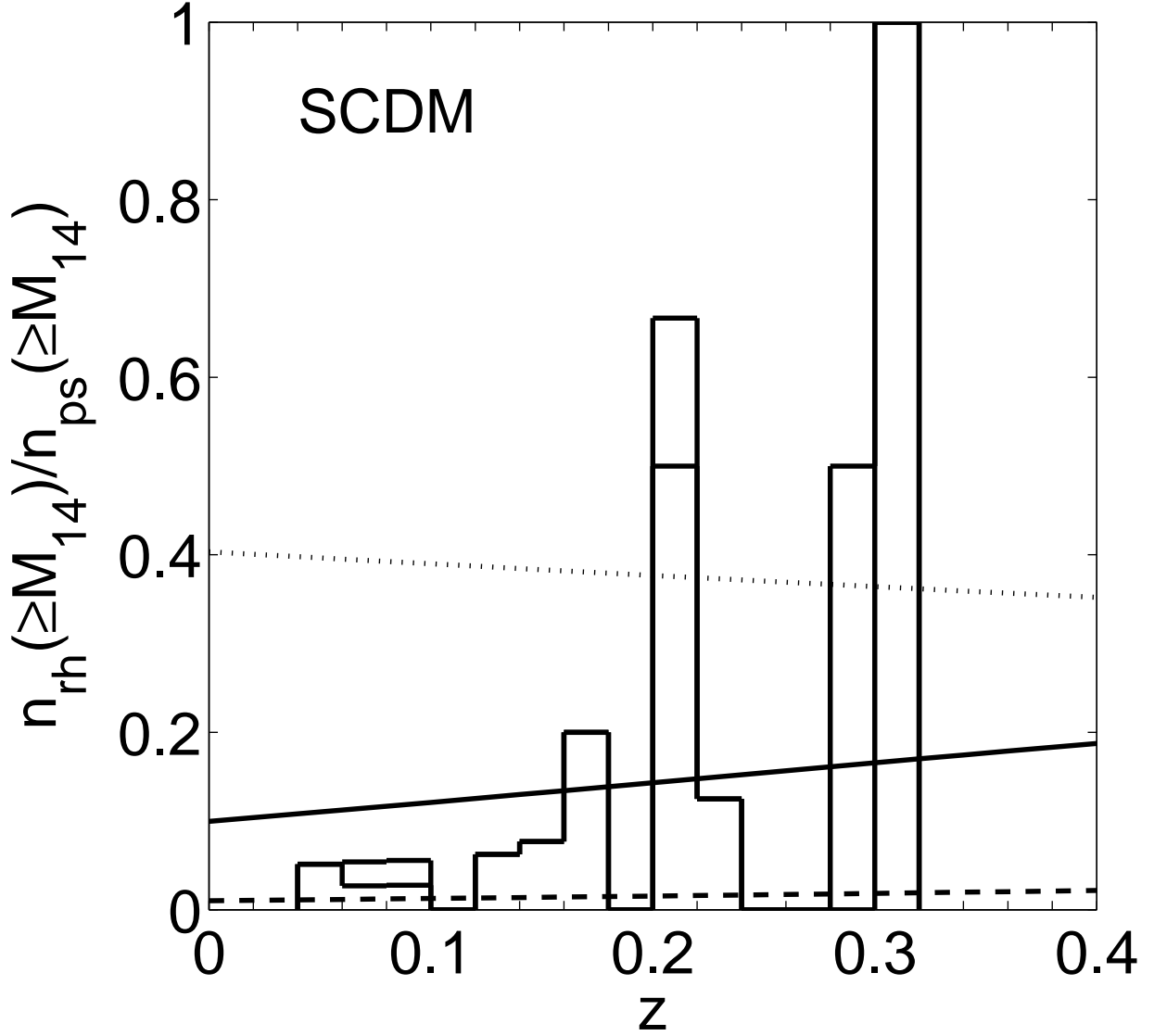


Fig. 3.— The distributions of the ratios of the total number density of radio halos to that of galaxy clusters in the SCDM model. The results for three representative lifetimes of radio halos: 0.1 Gyr (*dashed curves*), 1 Gyr (*solid curves*), and the cosmological time (*dotted curves*) are shown. The histograms show the observational results of Figure 2; the higher histogram includes the uncertain halos.

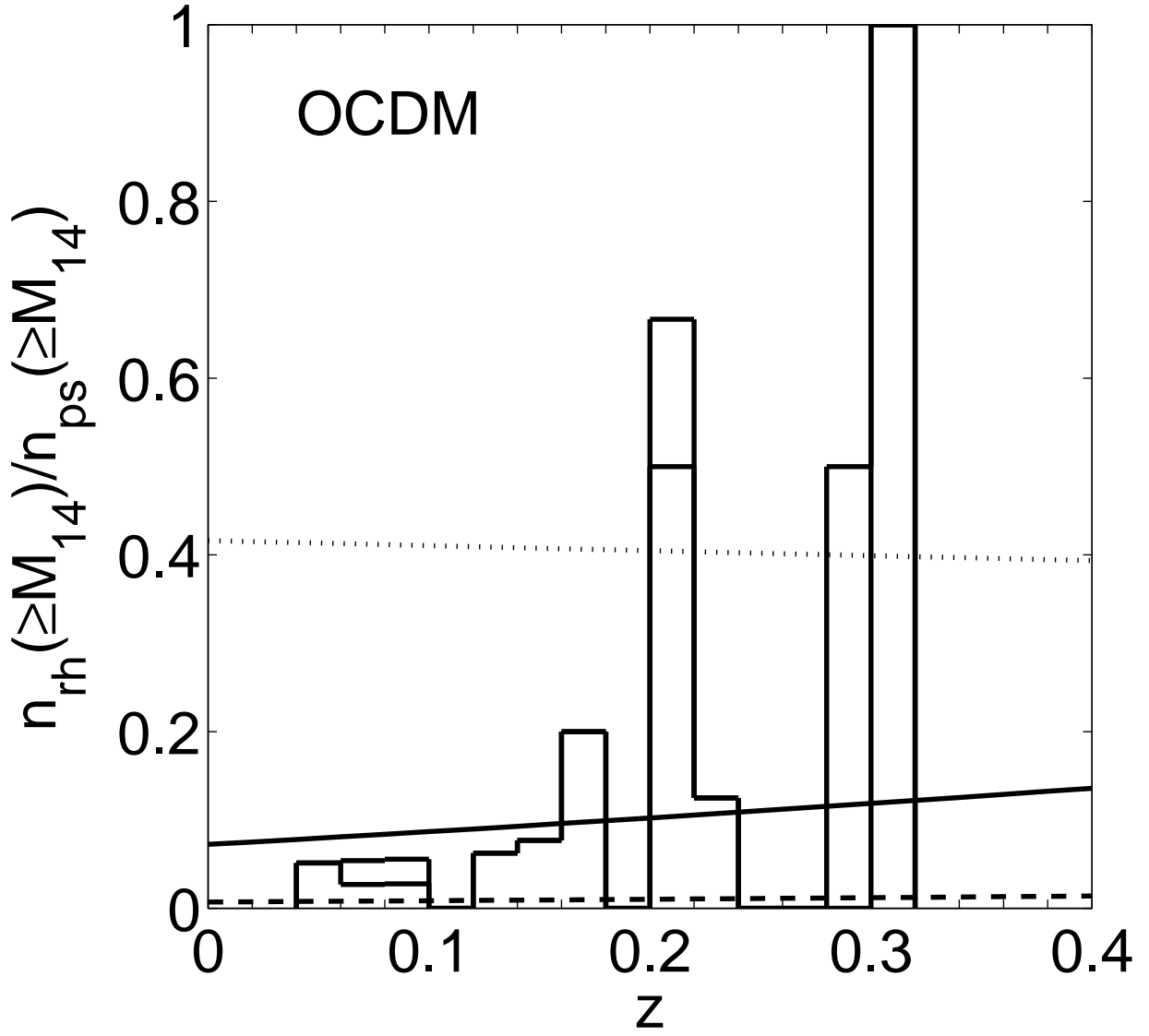


Fig. 4.— Same as Fig. 3, but for the OCDM model.

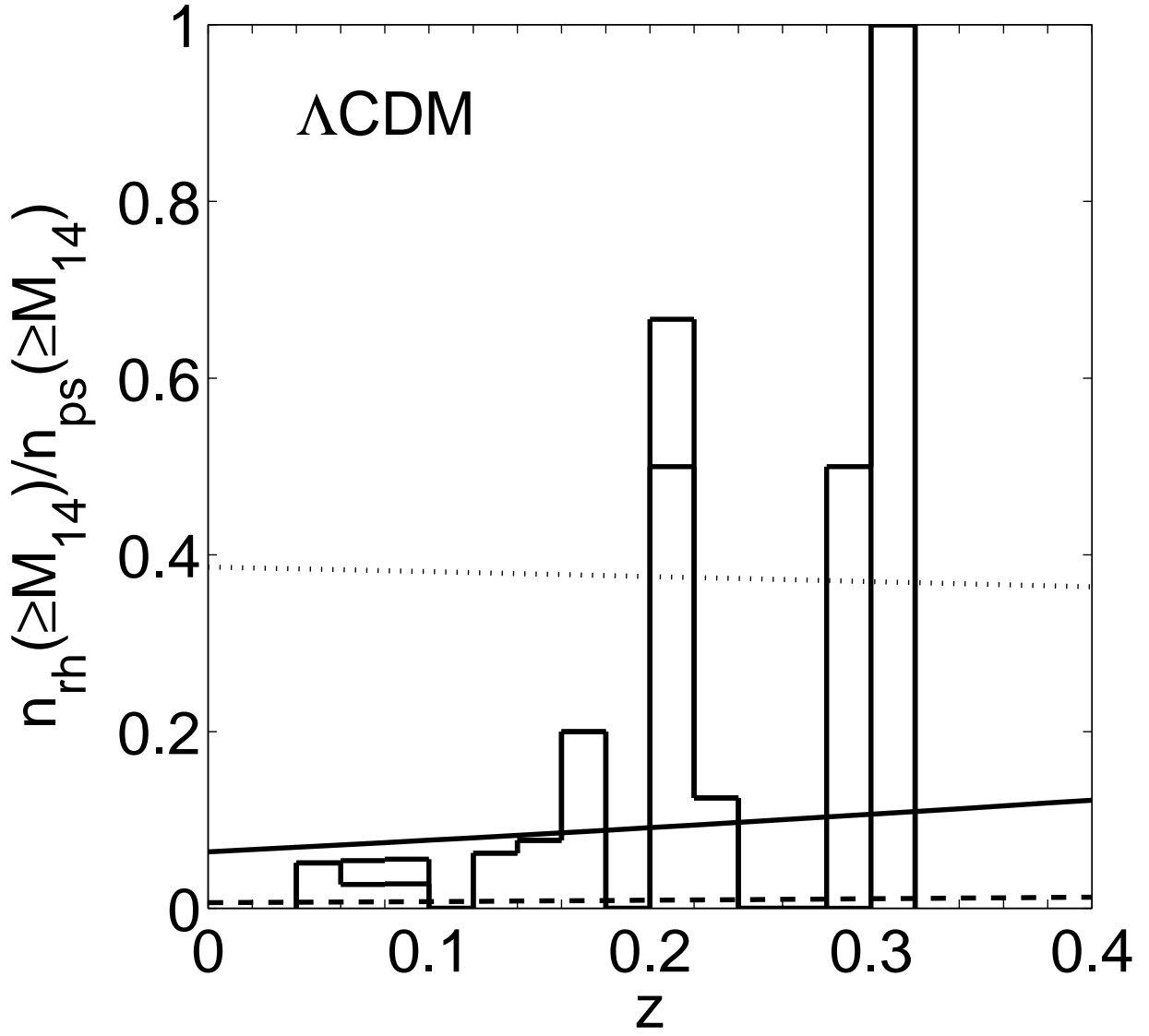


Fig. 5.— Same as Fig. 3, but for the Λ CDM model.

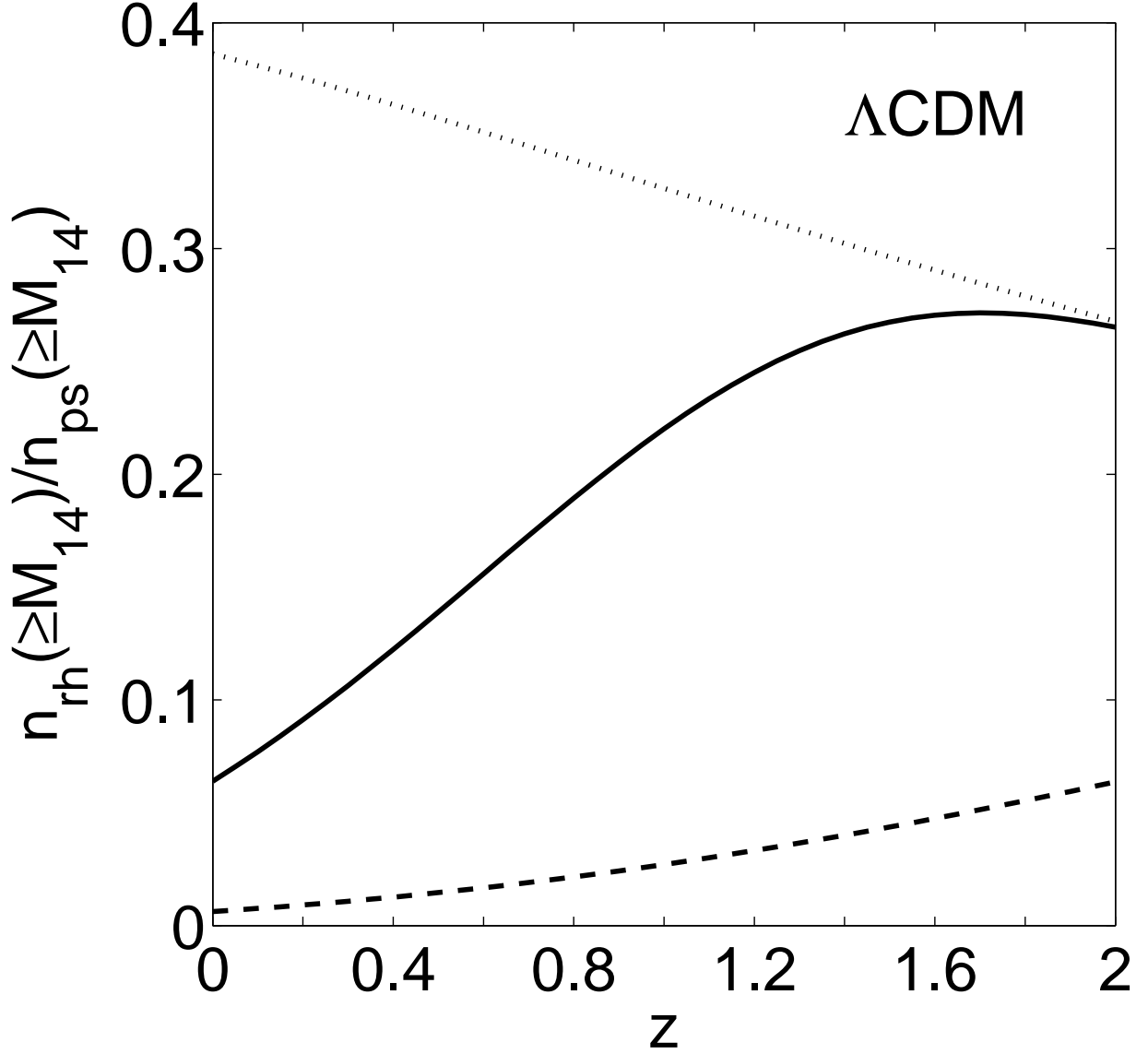


Fig. 6.— The evolution of the ratios of the total number density of radio halos to that of galaxy clusters in the ΛCDM model. The results for three representative lifetimes of radio halos: 0.1 Gyr (*dashed curves*), 1 Gyr (*solid curves*), and the cosmological time (*dotted curves*) are shown.